Matter and Energy

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Nuclear energy is a typical example of the application of the equivalence between matter and energy. In nuclear fission and fusion reactions, when the total mass of the products is lower than that of the reactants, energy is released according to $E = mc^2$. This energy is released via photons, as explained by the Planck relation, $E = hf$, which is based on undeniable experimental evidence. Much has been discussed about conversion of energy to matter, but the nature of this process remains mysterious. This paper presents a theoretical study that shows that a system possessing electromagnetic energy has inertia corresponding to the relativistic equivalent (mass) on the basis of its momentum and the Doppler effect. When this system undergoes translation, its space energy changes. This phenomenon follows a wave standard similar to that observed one for free particles.

I. INTRODUCTION

The equivalence between matter and energy, $E = mc^2$, established by Einstein, explains how energy E can be obtained at the expense of mass m, for example, during the fission of heavy nuclei or fusion of light atoms, where c is the speed of light. The relation also explains the origin of nuclear binding energy—it is the energy equivalent of the *mass defect* between the mass of an atomic nucleus and the sum of the masses of its constituents. Today, mass-to-energy equivalence is an integral feature in understanding nuclear reactions. For massless particles such as photons, which can be absorbed or emitted during these reactions causing a corresponding increase or decrease in inertia, respectively, the Planck relation, $E = hf$, is observed, where h is Planck's constant and f is the frequency.

Despite the fact that the equivalence between matter and energy is well defined, the conversion process is still unclear. Another issue is the uncertainty regarding the origin of inertia or of matter itself [1]. Under these circumstances, it appears appropriate to conduct a study of the energy retained in the stationary state by a specific system. Here, first, an argument is developed to determine the inertia for electromagnetic energy retained as a stationary wave. Then, the motion state for the same system is established through wave analysis. This is accomplished using well-established principles [2, 3].

In classical mechanics, the mass of a material particle is considered to be equivalent to its inertia, i.e., the difficulty in changing its state of motion. The mass m and velocity $\mathbf{v}$ determine the momentum p of the particle, as $\mathbf{p} = m\mathbf{v}$. For electromagnetic waves, momentum $p = E/c$. In microscopic terms, a wave is associated with every matter particle, and its momentum is established as $|\mathbf{p}| = h/\lambda$.

The system considered in this study is a mirror box containing, without any loss of energy, an electromagnetic wave

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of energy E. For simplicity, this box is assumed to be one-dimensional and correspondingly having a single axis.

II. INERTIA ATTRIBUTED TO ENERGY

Let us consider a system where two mirrors are placed facing each other at a distance d, and an electromagnetic wave is set up between these mirrors. The system is at rest and the stationary energy E is distributed uniformly over d in both the directions. In this case, there exist two waves with energy $E/2$: one with momentum p^+ propagating in one direction and the other with momentum p^- propagating in the opposite direction. Therefore,

$$p^+ = \frac{E}{2c} \qquad and \qquad p^- = -\frac{E}{2c} \qquad (1)$$

The resultant momentum is given by

$$p = p^+ + p^- = 0$$

Let E_c be the energy in each cycle of the electromagnetic wave. Then, $E_c = \rho_x \lambda$, where λ is the wavelength and $\rho_x = E/(2d)$ is the linear density of energy in each propagation direction. Thus, $E_c = E\lambda/(2d)$ and as $\lambda = c/f$, we get

$$E = \frac{2E_c d}{\lambda} = 2\frac{E_c d}{c}f \qquad (2)$$

Next, we consider that no energy loss has occurred to define a constant, i.e.,

$$\hat{h} = \frac{E_c d}{c} \qquad (3)$$

Then, Equation (2) can be expressed as

$$E = 2\hat{h}f \qquad (4)$$

Substituting this value for energy in Equation (1), we get

$$p^+ = \hat{h}\frac{f}{c} \qquad and \qquad p^- = -\hat{h}\frac{f}{c} \qquad (5)$$

Note that when dealing with energy quantization, $(E/2) = nhf$, where n is the number of identical photons, and so $\hat{h} = nh$.

Next, consider an observer on the right side of the mirror system, which we define as the positive direction. Then, if this observer moves with non-relativistic velocity $\mathbf{v}$ in this direction, there will be a frequency shift resulting in two new frequencies, f^+ for the wave propagating in the observer's direction and f^- for the other. This shift can be expressed as follows on the basis of the Doppler effect:

$$f^{\pm} = f\left(1 \pm \frac{v}{c}\right) \qquad (6)$$

Thus,

$$p'^+ = \hat{h}\frac{f}{c}\left(1 + \frac{v}{c}\right) \qquad and \qquad p'^- = -\hat{h}\frac{f}{c}\left(1 - \frac{v}{c}\right) \qquad (7)$$

In this case, the resultant momentum is $p' = p'^+ + p'^-$. Thus, through substitution and simple algebraic operations, we get

$$p' = \frac{2\hat{h}f}{c^2}\mathbf{v} \qquad (8)$$

The term in the numerator in the above equation corresponds to the energy expressed in Equation (4). Thus, by substituting Equation (4) in the above equation, we get

$$p' = \frac{E}{c^2}\mathbf{v} \qquad (9)$$

On comparing Equation (9) with $p = m\mathbf{v}$, we conclude that this system shows inertia equal to the mass $m = E/c^2$, which corresponds to Einstein's equation, $E = mc^2$.

III. WAVE ANALYSIS AND MOTION STATE

A. Waves in Stationary System

The energy of an electromagnetic wave is contained in electric and magnetic fields that are perpendicular to each other. In this study, we considered a generic plane wave ψ expressed as follows:

$$\psi = A\cos(kx - \omega t) \qquad (10)$$

This function could be used to express the intensity of either the electric or magnetic field in terms of propagation distance x and time t. In this case, it is considered to represent the electric field, where A is the maximum amplitude and the change in the propagation direction upon reflection of the plane wave results in phase inversion. Then, for propagation in the positive direction, the above equation becomes

$$\psi^+ = A\cos(k_0 x - \omega_0 t) \qquad (11)$$

and for propagation in the negative direction, it becomes

$$\psi^- = -A\cos((-k_0)x - \omega_0 t) \qquad (12)$$

The subscript "0" represents the system state, i.e., the stationary state of the mirror box. The distance d between the mirrors is selected such that begins at $x = 0$ and $2d$ is an integer multiple of λ_0. Within the interval bounded by the reflexion points, the overlapping of these waves is described by algebraically adding Equations (11) and (12):

$$\Psi = A\cos(k_0 x - \omega_0 t) - A\cos((-k_0)x - \omega_0 t) \qquad (13)$$

Using the trigonometric identity, Equation (13) can be expressed as

$$\Psi = -2\sin(k_0 x)\sin(-\omega_0 t) \qquad (14)$$

Since $k_0 = 2\pi/\lambda_0$, the points at $x = n\lambda_0/2$ behave as nodes (zero amplitude), and between these nodes exist antinodes whose amplitudes vary with time and have frequency $f_0 = 2\pi/\omega_0$. Figure 1 schematically illustrates this behavior.
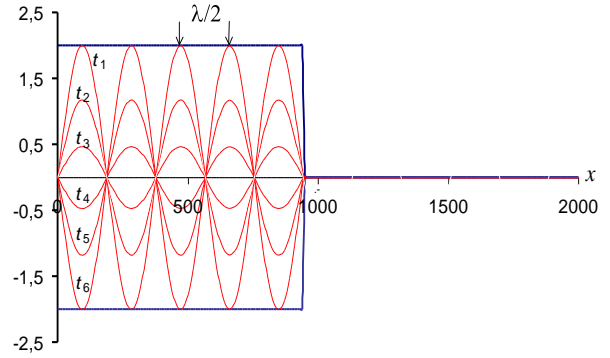

Figure 1: Stationary wave present in a stationary one-dimensional box. The amplitudes obtained from times $t = 1$ to $t = 6$ are superimposed in this figure. Here, $d = n\lambda/2$

B. Waves in System under Translation

If the box is moved at velocity $\mathbf{v}$, the frequency changes on account of the Doppler effect to f^+ in the direction of movement and f^- in the opposite direction, as obtained in Equation (6). In this case, the angular speeds, wave numbers, and wavelengths are expressed as follows:

$$\omega^+ = 2\pi f^+ \qquad and \qquad k^+ = \frac{2\pi}{\lambda^+} = 2\pi f^+/c \qquad (15)$$

$$\omega^- = 2\pi f^- \qquad and \qquad k^- = \frac{2\pi}{\lambda^-} = 2\pi f^-/c \qquad (16)$$

Substituting these equations into Equation (13), we get

$$\Psi = A\cos(k^+ x - \omega^+ t) - A\cos(k^- x - \omega^- t) \qquad (17)$$

Equation (17) can be put in product form with the variables arranged to get

$$\Psi = -2A \ \sin\{(1/2)[(k^+ + k^-)x - (\omega^+ + \omega^-)t)]\} \\ \times \sin\{(1/2)[(k^+ - k^-)x - (\omega^+ - \omega^-)t)]\} \ (18)$$

The wave number k is vectorial, where the direction of k^- is opposite to that of k^+. Substituting Equation (6) in the equations of ω^+ and ω^-, i.e., Equations (15) and (16), and then substituting these in the above equation, we get

$$\Psi = -2A \sin[(k_0 v/c)x - \omega_0 t] \sin[k_0 x - (\omega_0 v/c)t] \quad (19)$$

Figure 2 shows a snapshot of the wave (solid red line) resulting from the multiplication of two sinusoidal waves (stippled black lines).
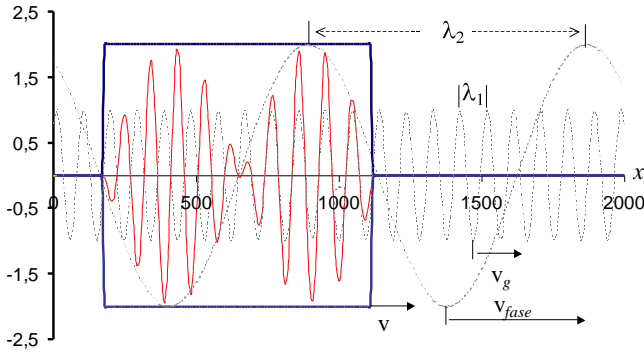

Figure 2: Snapshot of wave Ψ obtained by the multiplication of two sinusoidal waves, one with length λ_1 and velocity v_g and the other with length λ_2 and velocity v_{phase}. The velocity of the mirror box containing the wave is **v**.

We make a brief observation about Figure 2 using Equation (19). The last sine term in the equation represents a wave with frequency $f_g = (\omega_0 v)/(2\pi c)$ or $f_g = (v/c)f_0$. The propagation speed v_g is expressed as

$$v_g = \frac{\omega_0 v/c}{k_0}$$

Substituting $k_0 = \omega_0/c$ in the above equation, we get

$$v_g = \frac{\omega_0 v/c}{\omega_0/c} = v$$

Therefore, the nodes are fixed relative to the box and they are displaced at velocity v_g equal to the velocity of the box.

The other term in Equation (19), i.e., $sin[(k_0 v/c)x - \omega_0 t]$, has frequency $f_0 = 2\pi/\omega_0$ and wavelength λ_2 such that

$$\lambda_2 = \frac{2\pi}{k_0 v/c} = \frac{2\pi}{\omega_0 v/c^2} = \frac{2\pi}{2\pi f_0 v/c^2} = \frac{c}{(v/c)f_0}$$

Then, substituting $f_g = (v/c)f_0$, we obtain $\lambda_2 = c/f_g$. The propagation speed v_{phase} is given by

$$v_{phase} = \frac{\omega_0}{k_0 v/c} = \frac{c}{v}c$$

Here, it can be observed that $v \to 0 \Rightarrow v_{phase} \to \infty$.

The energy density in the electric and magnetic fields are, respectively,

$$U_E = \frac{1}{2}\varepsilon_0 \mathbf{E}^2 \qquad and \qquad U_B = \frac{1}{2}\frac{1}{\mu_0}\mathbf{B}^2$$

Owing to this, if the wave function in Equation (19) is squared, the resultant term will be proportional to the energy (function of time t and position x) regardless of whether the field is magnetic or electric. Therefore, $U_E(x,t) = K\Psi^2$, where

$$\Psi^2 = 4A^2 \sin^2[(k_0 v/c)x - \omega_0 t] \sin^2[k_0 x - (\omega_0 v/c)t] \quad (20)$$
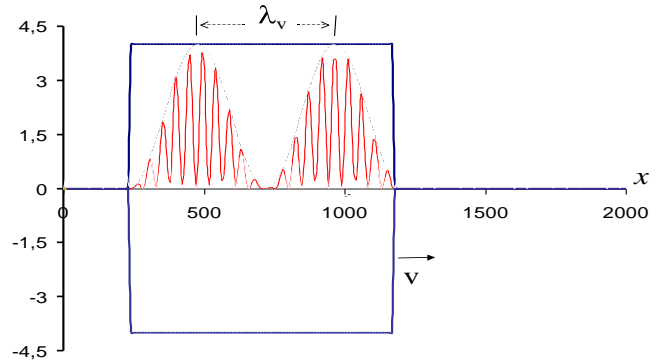

Figure 3: Effect of motion of the box containing the wave: this is visible as density variation along the x axis in the wave pattern with wavelength λ_v

Again, using the trigonometric identity, we can write Equation (20) as

$$\Psi^2 = 4A^2(1/2)\{1 - \cos[(k_0 2v/c)x - 2\omega_0 t]\} \\ \times (1/2)\{1 - \cos[2k_0 x - (\omega_0 2v/c)t]\} \quad (21)$$

A snapshot of Ψ^2 is shown in Figure 3. It is observed that in function of the translation movement, the energy U_E along the x axis is presented in a modulation wave standard with wavelength λ_v, proceeding from the first cosine function in Equation (21).

The wave number, k_v, is expressed as a function of velocity as below:

$$k_v = k_0 \frac{2v}{c}$$

Then, it follows that

$$\lambda_v = \frac{2\pi}{k_v} = \frac{2\pi}{k_0}\frac{c}{2v} = \lambda_0\frac{c}{2v}$$

$$\lambda_v = \frac{c^2}{f_0 2v}$$

Using equation (8), where

$$p' = \frac{2\hat{h}f}{c^2}v$$

the linear momentum can be obtained as

$$p = \frac{\hat{h}}{\lambda_v}$$

For *n* photons, $\hat{h} = nh$; then, $p = nh/\lambda_v$, which represents the correspondence between λ_v and the de Broglie wavelength.

IV. CONCLUSION

This theoretical study demonstrated that a system having electromagnetic energy shows inertia corresponding to the relativistic equivalent (mass) on the basis of its momentum and the Doppler Effect. Under translation, the same system shows a modulation in its energy along the direction of translation. This phenomenon corresponds to a wave standard similar to that observed for matter particles in the Schrödinger equation for free particles.

These results provide new perspectives in unraveling the mysteries of nature that are described by modern theories, in the light of old theories. The results also open up avenues for a relationship between relativity and quantum mechanics. This fact can be viewed as a potential research topic.

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